Unit Digits, Exponents, & Remainder Problems Solutions

1. There are several ways to deal with this problems some easier some harder, but almost all of them are based on the pattern recognition.

   The tens digit of 6 in integer power starting from 2 (6^1 has no tens digit) repeats in pattern of 5: {3, 1, 9, 7, 5}:
   The tens digit of 6^2=36 is 3;
   The tens digit of 6^3=216 is 1;
   The tens digit of 6^4=...96 is 9 (how to calculate: multiply 16 by 6 to get ...96 as the last two digits);
   The tens digit of 6^5=...76 is 7 (how to calculate: multiply 96 by 6 to get ...76 as the last two digit);
   The tens digit of 6^6=...56 is 5 (how to calculate: multiply 76 by 6 to get ...56 as the last two digits);
   The tens digit of 6^7=...36 is 3 again (how to calculate: multiply 56 by 6 to get ...36 as the last two digits).

   Hence, 6^2, 6^7, 6^12, 6^17, 6^22, ... will have the same tens digit of 3.

   Answer: B.

2. If we look at the power of 7, it shows a repeated trend at the Unit's digit. For e.g.:
   7^1 = 7 (7 at unit place)
   7^2 = 49 (9 at unit place)
   7^3 = 343 (3 at unit place)
   7^4 = 2401 (1 at unit place)
   7^5 = 16807 (7 at unit place)

   So if you see, this trend of 7,9,3,1......7,9,3,1.......repeats itself.
   7^131 will give 3 as unit's digit and when it is divided by 5, it will give the remainder as 3.
   So the answer is D.

3. The units digit of 3 in positive integer power has cyclicity of 4 for the unis digit:
   3^1 --> the units digit is 3;
   3^2 --> the units digit is 9;
   3^3 --> the units digit is 7;
   3^4 --> the units digit is 1;
   3^5 --> the units digit is 3 AGAIN;
   ...

   So, the units digit repeats the following pattern {3-9-7-1}-{3-9-7-1}-.... 3^{8n+3} will have the same units digit as 3^3, which is 7 (remainder when 8n+3 divided by cyclicity 4 is 3). Thus the last digit of $3^{8n+3}+2$ will be $7+2=9$. Any positive integer with the unis digit of 9 divided by 5 gives the remainder of 4.

Answer: E.

4. 7 in power repeats pattern of 4: 7-9-3-1. As 75=4*18+3 then the last digit of 7^{75} is the same as the last digit of 7^3, which is 3. Units digit of $7^{75}+6$ will be: 3 plus 6 = 9.

   Answer: E.

5. 3 in power has cyclicity of 4:
   1. 3^1=3 (last digit is 3)
   2. 3^2=9 (last digit is 9)
   3. 3^3=27 (last digit is 7)
   4. 3^4=81 (last digit is 1)
   5. 3^5=243 (last digit is 3 again!)
   ...

   To find the last digit of 3^{8n+3}, divide the power (which is $8n+3$) by cyclicity # (which is 4) and look at the remainder --> $\frac{8n+3}{4}$ --> $remainder=3$, which means that the last digit of 3^{8n+3} will be the same as the last digit of $3^3=27$ (last digit is 7). (Side note: If the remainder were 0, then last digit would be the same as the las digit of 3^4).

   Now, last digit of $3^{8n+3}+2$ will be $7+2=9$. Any integer with last digit 9 upon division by 5 yields remainder of 4.

   Answer: E.

6. The units digit of 73^350 will be the same as the units digit of 3^350.

   3^1=3 --> the units digit is 3;
   3^2=9 --> the units digit is 9;
   3^3=27 --> the units digit is 7;
   3^4=81 --> the units digit is 1;
   3^5=243 --> the units digit is 3 AGAIN;
   ...

   So, as you can see the units digit repeats in blocks of 4: {3, 9, 7, 1}, {3, 9, 7, 1}, ... Now, since 350=348+2=(multiple of 4)+2, then the units digit of 3^350 will be the second number in the pattern thus 9.

   Answer: E.

7. The unit's digit of 7 in positive integer power repeats in blocks of 4: {7-9-3-1}. Since 75=4*18+3 then the unit's digit of 7^{75} is the same as the unit's digit of 7^3, which is 3.

   Therefore the unit's digit of $7^{75}+6$ will be: 3 plus 6 = 9.

   Answer: E.

8. $51^{25}=(52-1)^{25}$, now if we expand this expression all terms but the last one will have $52=13*4$ in them, thus will leave no remainder upon division by 13, the last term will be $(-1)^{25}=-1$. Thus the question becomes: what is the remainder upon division -1 by 13? The answer to this question is 12: $-1=13*(-1)+12$.

   Answer: A.

9. 3^x will have a pattern with unit digit as 3, 9, 7,1, 3,9,7,1,....

   Thus, after every four values, the unit digit pattern will repeat.

   Divide 27 by 4....3 is the remainder. Unit digit for the third value (from the pattern above) will be 7.

10. Let's check which of the options is NOT a prime:

    A. 2^16 + 1 --> the units digit of 2 in positive integer power repeats in blocks of four {2, 4, 8, 6}. Hence, the units digit of 2^16 is 6 and the units digit of 2^16 + 1 is 7 --> 2^16 + 1 CAN be a prime.

    B. 2^31 + 3^31 --> the units digit of 2^31 is 8 and the units digit of 3^31 is 7 (the units digit of 3 in positive integer power repeats in blocks of four {3, 9, 7, 1}). Hence, the units digit of 2^31 + 3^31 is 5 (8+7). Thus 2^31 + 3^31 is divisible by 5. Not a prime.

    C. 4^66 + 7^66 --> the units digit of 4^66 is 6 (the units digit of 4 in positive integer power repeats in blocks of two {4, 6}) and the units digit of 7^66 is 9 (the units digit of 7 in positive integer power repeats in blocks of four {7, 9, 3, 1}). Hence, the units digit of 4^66 + 7^66 is 5 (6+9). Thus 4^66 + 7^66 is divisible by 5. Not a prime.

    D. 5^82 - 2^82 --> we can factor this as (5^41 - 2^41)(5^41 + 2^41). Not a prime.

    E. 5^2881+ 7^2881 --> 5^2881 + 7^2881 = odd + odd = even. Not a prime.

    Only option A can be prime.

Answer: A.

11. Notice that $43^{86}=(40+3)^{86}$. Now, if we expand this expression, all terms but the last one will have 40 as multiple and thus will be divisible by 5. The last term will be 3^{86}. So we should find the remainder when 3^{86} is divided by 5.

    Next, $3^{86}=9^{43}$. 9 in odd power has units digit of 9 hence yields the remainder of 4 upon division by 5 (9 in even power has units digit of 1 hence yields the remainder of 1 upon division by 5).

    Answer: E.

12. The remainder when 43717^(43628232) is divide by 5 will be the same as the remainder when 7^(43628232) is divided by 5 (we need only the units digit to get the remainder upon division by 5).

    7^1=7 divided by 5 yields the remainder of 2;
    7^2=49 divided by 5 yields the remainder of 4;
    7^3=343 divided by 5 yields the remainder of 3;
    7^4=...1 divided by 5 yields the remainder of 1.
    7^5=...7 divided by 5 yields the remainder of 2 AGAIN.

    The remainders repeat in blocks of four {2, 4, 3, 1}, {2, 4, 3, 1}, ...

    43628232 (exponent) is divisible by 4 (a number is divisible by 4 if its last 2 digits (32 in our case) divisible by 4). Therefore, the remainder when 43717^(43628232) is divided by 5 is the fourth number in pattern, which is 1.

    Answer: A.

13. Just follow the unit digits:

    n = (33)^43 + (43)^33

    Here the last digits would determine the power.

So,

n = (33)^43 + (43)^33 ~ 3^43 + 3 ^33

Now last digits of 3^1, 2, 3, 4, 5 = 3, 9, 7, 1, 3. The cycle repeats after every 4 rounds.

So n = 3^43 + 3 ^33 = 3^(40+3) + 3^(32+1) = {(3^(4*10)}{3^3} + {(3^(4*8)}{3^1}

Now last digits of these terms would be {1}{7} + {1}{3} = 10

Hence the last digit is 0. Answer is A.

14. The units digit of 2222^{333} is the same as that of 2^{333};
The units digit of 3333^{222} is the same as that of 3^{222};
Hence, the units digit of $2222^{333}*333^{222}$ is the same as that of $2^{333}*3^{222}$;

    Now, the units digits of both 2 and 3 in positive integer power repeat in patterns of 4. For 2 it's {2, 4, 8, 6} and for 3 it's {3, 9, 7, 1}.

    The units digit of 2^{333} will be the same as that of 2^1, so 2 (as 333 divided by cyclicity of 4 yields remainder of 1, which means that the units digit is first # from pattern);
The units digit of 3^{222} will be the same as that of 3^2, so 9 (as 222 divided by cyclicity of 4 yields remainder of 2, which means that the units digit is second # from pattern);

    Finally, 2*9=18 --> the units digit is 8.

    Answer: E.

15. First of all I think that this question is a little bit out of the scope of the GMAT. But anyway:

    The last digit of 3 in positive integer power repeats in pattern of 4: {3, 9, 7, 1}. So, basically we should find the remainder upon division 7^(11) by cyclicity of 4 (to see on which number in this pattern 7^{11} falls on). $7^{11}=(4+3)^{11}$, now if we expand this expression all terms but the last one will have 4 in them, thus will leave no remainder upon division by 4, the last term will be 3^{11}. Thus the question becomes: what is the remainder upon division 3^{11} by 4:
3 divided by 4 yields remainder of 3;
3^2=9 divided by 4 yields remainder of 1;
3^3=27 divided by 4 yields remainder of 3;
3^4=81 divided by 4 yields remainder of 1.

    So, 3 in odd power yields remainder of 1 upon division by 4 --> 3^{11} yields remainder of 3

--> finally, we have that $3^{7^{11}}$ will have the same last digit as 3^3, which is 7. Thus as $3^{7^{11}}$ has the last digit of 7 then divided by 5 it will yield remainder of 2.

Answer: C.

16. 32^32^32 = (28+4)^32^32
As 28 is divisible by 7, we dont need to worry about that part. Hence for the purpose of remainder,
our equation boils down to 4^32^32

The cyclicity of 4 is 3 when divided by 7, hence we need to think about the value of 32^32 and what remainder it leaves when divided by 3.

Considering 32^32, it can be broken into (30+2)^32. Again 30^32 is divisible by 3. Hence we need to focus on 2^32.
2^32 can be written as (2*2)^31 = (3+1)^31. As 3^31 is also divisible by 3, we will be left with 1^31.
Thus 1 would be the remainder when 32^32 is divided by 3.

This implies that 4 will be the remainder when divided by 7.
Hence Answer is B.

17. Must know for the GMAT:
I. The units digit of $(abc)^n$ is the same as that of c^n, which means that the units digit of $(17^3)^4$ is that same as that of $(7^3)^4$ and the units digit of 1973^{3^2} is that same as that of 3^{3^2}.

II. If exponentiation is indicated by stacked symbols, the rule is to work from the top down, thus:
$a^{m^n}=a^{(m^n)}$ and not $(a^m)^n$, which on the other hand equals to a^{mn}.

So:
$(a^m)^n=a^{mn}$;

$a^{m^n}=a^{(m^n)}$.

Thus, $(7^3)^4=7^{(3*4)}=7^{12}$ and $3^{3^2}=3^{(3^2)}=3^9$.

III. The units digit of integers in positive integer power repeats in specific pattern (cyclicity): The units digit of 7 and 3 in positive integer power repeats in patterns of 4:

1. 7^1=7 (last digit is 7)
2. 7^2=9 (last digit is 9)
3. 7^3=3 (last digit is 3)
4. 7^4=1 (last digit is 1)
5. 7^5=7 (last digit is 7 again!)
...

1. 3^1=3 (last digit is 3)
2. 3^2=9 (last digit is 9)
3. 3^3=27 (last digit is 7)
4. 3^4=81 (last digit is 1)
5. 3^5=243 (last digit is 3 again!)
...

Thus th units digit of 7^{12} will be 1 (4th in pattern, as 12 is a multiple of cyclicty number 4) and the units digit of 3^9 will be 3 (first in pattern, as 9=4*2+1).

So, we have that the units digit of $(17^3)^4=17^{12}$ is 1 and the units digit of $1973^{32}=1973^9$ is 3. Also notice that the second number is much larger then the first one, thustheir difference will be negative, something like 11-13=-2, which gives the final answer that the units digit of $(17^3)^4-1973^{32}$ is 2.

Answer B.

18. $333^{222}=(329+4)^{222}=(7*47+4)^{222}$. Now if we expand this, all terms but the last one will have 7*47 as a multiple and thus will be divisible by 7. The last term will be $4^{222}=2^{444}$. So we should find the remainder when 2^{444} is divided by 7.

2^1 divided by 7 yields remainder of 2;
2^2 divided by 7 yields remainder of 4;
2^3 divided by 7 yields remainder of 1;

2^4 divided by 7 yields remainder of 2;
2^5 divided by 7 yields remainder of 4;
2^6 divided by 7 yields remainder of 1;
...

The remainder repeats in blocks of three: {2-4-1}. So, the remainder of 2^{444} divided by 7 would be the same as 2^3 divided by 7 (444 is a multiple of 3). 2^3 divided by 7 yields remainder of 1.

Answer: E.

19. This question is beyond the GMAT scope. It can be solved with Fermat's little theorem, which is not tested on GMAT. Or another way:

    $(18^{22})^{10}=18^{220}=(14+4)^{220}$ now if we expand this all terms but the last one will have 14 as multiple and thus will be divisible by 7. The last term will be 4^{220}. So we should find the remainder when 4^{220} is divided by 7.

    $4^{220}=2^{440}$.

    2^1 divided by 7 yields remainder of 2;
    2^2 divided by 7 yields remainder of 4;
    2^3 divided by 7 yields remainder of 1;

    2^4 divided by 7 yields remainder of 2;
    2^5 divided by 7 yields remainder of 4;
    2^6 divided by 7 yields remainder of 1;
    ...

    So the remainder repeats the pattern of 3: 2-4-1. So the remainder of 2^{440} divided by 7 would be the same as 2^2 divided by 7 (440=146*3+2). 2^2 divided by 7 yields remainder of 4.

    Answer: D.

20. The last digit of 2^k repeats in pattern of 4 (cyclicity is 4):
    2^1=2 --> last digit is 2;
    2^2=4 --> last digit is 4;
    2^3=8 --> last digit is 8;
    2^4=16 --> last digit is 6;
    2^5=32 --> last digit is 2 again;

    Now, when k itself is a multiple of 4 (when there is no remainder upon division k by cyclicity number), then the last digit will be the last digit of 2^4 (4th in pattern), so 6 not 1 (taking 2^0) as you've written.

    If k is a positive integer, what is the remainder when 2^k is divided by 10?

    Notice that all we need to know to answer the question is the last digit of 2^k.

    (1) k is divisible by 10 --> different multiples of 10 yield different remainders upon division by 4 (for example 10/4 yields 2 and 20/4 yields 0), thus we can not get the single numerical value of the last digit of 2^k. Not sufficient.

(2) k is divisible by 4 --> as discussed, when k is a multiple of 4, the last digit of 2^k equals to the last digit of 2^4, which is 6. Integer ending with 6 yields remainder of 6 upon division by 10. Sufficient.

Answer: B.

21. Last digit of 3^x repeats in blocks of 4: {3, 9, 7, 1} - {3, 9, 7, 1} - ... So cyclicity of the last digit of 3 in power is 4. Now, 3^{4n+2} will have the same last digit as 3^2 (remainder upon division 4n+2 upon cyclicity 4 is 2, which means that 3^{4n+2} will have the same last digit as 3^2). Last digit of 3^2 is 9. So $3^{4n+2}+1$ will have the last digit $9+1=0$. Number ending with 0 is divisible by 10 (remainder 0). Sufficient.

    (2) x > 4. Clearly insufficient.

    Answer: A.

22. The units digit of 243^x is the same as the units digit of 3^x and similarly the units digit of 463^y is the same as the units digit of 3^y, so the units digit of 243^x*463^y equals to the units digit of $3^x*3^y=3^{x+y}$. So, knowing the value of $x+y$ is sufficient to determine the units digit of n.

    (1) $x+y=7$. Sufficient. (As cyclicity of units digit of 3 in integer power is 4, units digit of 3^7 would be the same as of units digit of 3^3 which is 7)

    (2) $x=4$. No info about y. Not sufficient. Answer is A.

23. First of all, when a positive integer is divided by 10, the remainder is the units digit of that integer. For example, 30 divided by 10 yields the remainder of 0, 31 divided by 10 yields the remainder of 1, 32 divided by 10 yields the remainder of 2, ...

    Next, the units digit of 2 in positive integer power repeats in blocks of 4: {2, 4, 8, 6}

    The units digit of 2^1 is 2;
    The units digit of 2^2 is 4;
    The units digit of 2^3 is 8;
    The units digit of 2^4 is 6;
    The units digit of 2^5 is 2, AGAIN;
    ...

    (1) s is even --> rst is even, hence the units digit of 2^(rst) is either 4 or 6. Not sufficient.

(2) rs = 4 --> rst is a multiple of 4, hence the units digit of 2^(rst) is the same as the units digit of 2^4 so 6, which means that the remainder upon division of 2^(rst) by 10 is 6. Sufficient.

Answer: B.

24. (1) $100<y^2<x^2<169$ --> since both x and y are positive integers then x^2 and y^2 are perfect squares --> there are only two perfect squares in the given range 121=11^2 and 144=12^2 --> $y=11$ and $x=12$. Sufficient.(As cyclicity of units digit of 7 in integer power is 4, therefore the units digit of 7^{23} is the same as the units digit of 7^3, so 3).

(2) $x^2-y^2=23$ --> $(x-y)(x+y)=23=prime$ --> since both x and y are positive integers then: $x-y=1$ and $x+y=23$ --> $y=11$ and $x=12$. Sufficient.

Answer: D.

25. Last digit of $3^{positive\ integer}$ repeats in blocks of 4: {3, 9, 7, 1} - {3, 9, 7, 1} - ... So cyclicity of the last digit of 3 in power is 4. Now, $3^{4+4x}=3^{4(1+x)}$ will have the same last digit as 3^4 which is 1 (remainder upon division the power 4+4x by cyclicity 4 is 0, which means that $3^{(4+4x)}$ will have the same last digit as 3^4).

Now $3^{(4+4x)}+9^y$ will be divisible by 10 if $y=odd$, in this case the last digit of 9^{odd} will be 9 (9 has a cyclicity of 2: {9, 1} - {9, 1} - ...) so the last digit of $3^{(4+4x)}+9^y$ will be 1+9=0.

(1) x = 25. Not sufficient.
(2) y = 1 --> $y=odd$ -> remainder=0. Sufficient. Answer B.